

# Instructions for Reinterpretation

We provide signal efficiency parameterization for the cluster-level selections that allows for reproduction of the full-simulation signal yield for various LLP masses (7-55 GeV), lifetimes (0.1 - 100 m) and decay modes ( $d\bar{d}$  and  $\tau^+\tau^-$ ). We provide the efficiency parameterization for the signal yield for bin D (signal region) of the ABCD plane, as defined in the paper. The signal contribution in the background-dominated bins A, B, and C are negligible in all of the considered signal samples. The signal generator cards for  $H \rightarrow SS$  signal samples are attached in Additional Resources. The data yields in the 4 bins in the ABCD plane are provided in the paper. In order to recast this analysis, only the generator-level LLP hadronic energy, EM energy, and decay position are needed for signal yield prediction. The following selection efficiencies are needed to account for all cluster-level selections applied for the signal region (bin D) mentioned in the paper:

- Cluster efficiency, including the segment and rechit vetos, muon veto, time spread cut, and  $N_{hits} \geq 130$ . This efficiency is provided as a function LLP EM and hadronic energy in two separate LLP decay regions A and B, as shown in Additional Figure 7 a and b, respectively. The LLP decay regions, EM, and hadronic energy are defined in the following paragraphs.
- The jet veto, time cut, and  $\Delta\phi(p_T^{miss}, \text{cluster})$  cut efficiencies. These three cuts are not included in the cluster efficiency parameterization, because they are model dependent. However, the 3 quantities can be calculated accurately for each model using generator level information, so it is left up to the recaster to calculate the efficiency of the jet veto, time cut, and  $\Delta\phi(p_T^{miss}, \text{cluster})$  cut for the specific models.
- Cut-based ID efficiency. This is provided in the python file `cut_based_id.py` attached in Additional Resources. The function uses the LLP decay position to predict the average station of the cluster and the  $N_{station} > 1$  efficiency parameterization (Additional Figure 8).

The LLP EM and hadronic energy is assigned by matching stable (status 1) GenParticles that are produced 0.1 m from the LLP decay vertex. Neutral pions, electrons, and photons are assigned as EM energy. All other particles, except for neutrinos and muons, are assigned as hadronic energy. Neutrinos and muons are ignored because they do not produce showers in the muon system.

The LLP decay region is categorized into 2 regions. These 2 regions have qualitatively different behavior. Within each region, they have quantitatively similar behavior, so we will provide the efficiency parameterization for each region separately. Region A is defined as  $391 \text{ cm} < r < 695.5 \text{ cm}$  and  $400 \text{ cm} < |z| < 671 \text{ cm}$ . Region B is defined as  $671 \text{ cm} < |z| < 1100 \text{ cm}$ ,  $r < 695.5 \text{ cm}$ , and  $|\eta| < 2$ . The fraction of LLPs that decay in each region are dependent on the LLP mass and  $c\tau$ . However, the signal efficiency in B is much larger than A, so for the models considered, more than 90% of clusters passing all selections are from LLPs that decay in region B.

The cluster efficiency is parameterized in bins of LLP hadronic energy and EM energy in each LLP decay region (Additional Figure 7). The efficiency includes cluster-level selections mentioned in the paper, including the segment and rechit vetos, muon veto, time spread cut, and  $N_{hits} \geq 130$ , except for the jet veto, time cut, and  $\Delta\phi(p_T^{miss}, \text{cluster})$  cut. These latter requirements are model dependent, but they can be calculated accurately for each model using generator level information. When recasting the analysis, these additional selections need to be implemented, to be consistent with applying all selections described in the paper. The full simulation signal yield prediction for samples with various LLP mass between 7 - 55 GeV, lifetime between 0.1 - 100 m, and decay mode to  $d\bar{d}$  and  $\tau^+\tau^-$  can be reproduced using this parameterization to within 35% and 20% for region A and B, respectively.

The parameterization of the cut-based ID is provided in python file `cut_based_id.py` attached in Additional Resources. The function uses the LLP decay position to predict the average station of the cluster (`AvgStation` function in the code) and the  $N_{station} > 1$  efficiency parameterization (Additional Figure 8). As described in the paper, the cluster ID requirement applies different  $\eta$  cuts depending on the  $N_{station}$  and average station number. We need a parametrization of the efficiency of the  $N_{station} > 1$  requirement and a transfer

function that takes gen-level LLP decay position to RECO-level cluster average station (only for clusters with  $N_{station} = 1$ ). Since the entire region A is in an  $\eta$  region ( $|\eta| < 1.3$ ) that passes all the  $\eta$  selections used in the cut-based ID (tightest cut is  $|\eta| < 1.6$ ), the cut-based ID efficiency in this region is 100%. The focus of the parameterization will be on region B only. The efficiency of  $N_{station} > 1$  in region B is provided in bins of LLP hadronic energy (Additional Figure 8) and the efficiency is independent of the LLP EM energy. The average station transfer function is provided as the **AvgStation** function in the code attached. The full simulation signal yield prediction for samples with various LLP mass between 7 - 55 GeV, lifetime between 0.1 - 100 m, and decay mode to  $d\bar{d}$  and  $\tau^+\tau^-$  can be reproduced using this parameterization to within 10%.